Let G be a CFG for language L with variables V and start symbol S. Let N be a DFA for language R with states Q, transitions δ, initial state 0 and accepting states F. Both are over the same (terminal) alphabet Σ.

Define G_N for $L \cap R$ to have variables $Q \times V \times Q$. For each original rule

$$A \to w_0\ A^1\ w_1\ A^2 \cdots\ A^n\ w_n\,,$$

$n \geq 0$, $w_i \in \Sigma^*$ and $A^j \in V$, include in G_N all rules

$$_{r_0}A_{q_{n+1}} \to w_0\ {}_{q_1}A^1_{r_1}\ w_1\ {}_{q_2}A^2_{r_2}\ \cdots\ {}_{q_n}A^n_{r_n}\ w_n\,,$$

such that $\delta(r_k, w_k) = q_{k+1}$ for each $k = 0, \ldots, n$. Add rules $S \to {}_0S_f$ for all $f \in F$, and make S the start symbol of G_N.

In particular, for an original rule $A \to w$, $w \in \Sigma^*$, include all rules $_rA_q \to w$, such that $\delta(r, w) = q$. This clearly includes all rules $_qA_q \to \varepsilon$ for an original rule $A \to \varepsilon$.

By induction on the length of a derivation, one can prove that the language $L({}_rA_q)$ of G_N is the intersection $L_G(A) \cap L_N(r, \{q\})$ between words that are derivable from the variable A of G and words that obtained on paths from r to q in N.

For example, here is how to get palindromes over $\{\mathbf{a}, \mathbf{b}\}$ with an even number of **a**'s. Consider the palindrome-producing grammar

$$S \;\to\; \varepsilon \mid \mathbf{a} \mid \mathbf{b} \mid \mathbf{a}S\mathbf{a} \mid \mathbf{b}S\mathbf{b}$$

and the even-counting automaton

δ	**a**	**b**
0	1	0
1	0	1

where 0 is the start and accepting state. By the above construction, we get the following grammar for their intersection:

$$\begin{aligned}
{}_0S_0 &\;\to\; \varepsilon \mid \mathbf{b} \mid \mathbf{a}\ {}_1S_1\ \mathbf{a} \mid \mathbf{b}\ {}_0S_0\ \mathbf{b} \\
{}_0S_1 &\;\to\; \mathbf{a} \mid \mathbf{a}\ {}_1S_0\ \mathbf{a} \mid \mathbf{b}\ {}_0S_1\ \mathbf{b} \\
{}_1S_0 &\;\to\; \mathbf{a} \mid \mathbf{a}\ {}_0S_1\ \mathbf{a} \mid \mathbf{b}\ {}_1S_0\ \mathbf{b} \\
{}_1S_1 &\;\to\; \varepsilon \mid \mathbf{b} \mid \mathbf{a}\ {}_0S_0\ \mathbf{a} \mid \mathbf{b}\ {}_1S_1\ \mathbf{b}
\end{aligned}$$

The start symbol is ${}_0S_0$.

Since the middle two rules are unreachable, this simplifies to

$$\begin{aligned}
{}_0S_0 &\;\to\; \varepsilon \mid \mathbf{b} \mid \mathbf{a}\ {}_1S_1\ \mathbf{a} \mid \mathbf{b}\ {}_0S_0\ \mathbf{b} \\
{}_1S_1 &\;\to\; \varepsilon \mid \mathbf{b} \mid \mathbf{a}\ {}_0S_0\ \mathbf{a} \mid \mathbf{b}\ {}_1S_1\ \mathbf{b}
\end{aligned}$$

Since the two variables are equivalent (generate the same languages), this reduces to just

$$S \;\to\; \varepsilon \mid \mathbf{b} \mid \mathbf{a}S\mathbf{a} \mid \mathbf{b}S\mathbf{b}$$