

Exercise 1 - Computational Models - Spring 2010/11

Notation: We denote by $\#_\sigma(w)$ the number of times the word $\sigma \in \Sigma^*$ is a substring in the word $w \in \Sigma^*$.

1. Present a DFA that accepts each of the following languages over $\Sigma = \{0, 1\}$:
 - (a) Σ^*
 - (b) $\{0\}\{1\}^*$
 - (c) $\{\epsilon, 100\}$
 - (d) $\{w \mid w \text{ does not contain '0101'}\}$
2. Let L_n be the language of words over $\Sigma = \{0, 1\}$ such that the n th character from the end is 0, for any n .
 - (a) Present a DFA that accepts L_n . Give a formal description and not a drawing.
 - (b) Present an NFA that accepts L_n .
 - (c) Present a regular expression for L_n .
3. Let w be a binary string. Let x_1 be the number represented by w using the *MSB first* representation, and x_2 be the number represented by w using the *LSB first* representation. Prove that x_1 is divisible by 3 iff x_2 is divisible by 3. Use automata theory in your proof.
4. Present an NFA and convert it to a DFA for the following languages over $\Sigma = \{0, 1\}$:
 - (a) $\{w \mid w \text{ contains '00' or doesn't contain '101'}\}$
 - (b) $\{xy \mid \#_0(x) \bmod 2 = 0 \text{ and } \#_1(y) \bmod 2 = 1\}$
5. Present a regular expression for the following languages over $\Sigma = \{0, 1\}$:

- (a) $\{w \mid |w| \bmod 4 = 0\}$
 - (b) $\{w \mid w \text{ contains exactly three '1's}\}$
 - (c) The complement of $\mathcal{L}((1 \cup 01 \cup 001)^*(\epsilon \cup 0 \cup 00))$
6. Given that L is a regular language over some alphabet Σ , prove that the following languages are regular:
- (a) $\{xy \mid (x \in L) \text{ XOR } (y \in L)\}$
 - (b) $\{x_1x_2 \cdots x_k \mid x_1, \dots, x_k \in \Sigma \text{ and } \exists y_1, y_2, \dots, y_k \in \Sigma, x_1y_1x_2y_2 \cdots x_ky_k \in L\}$
 - (c) $\{xy \mid yx \in L\}$
7. Is $\{w \in \{0, 1\}^* \mid \#_{01}(w) = \#_{10}(w)\}$ regular? Justify.
8. Find a regular expression for the language of binary strings that represent numbers divisible by 3 (MSB first). Use the algorithm learnt in class, show all steps.