Exercise 2 - Computational Models - Spring 2011

Submission date and time: 27/3/2011, 15:00
Your assignment should be in box 311 before 15:00 !

Note1: We denote by $\#_\sigma(w)$ the number of times the word $\sigma \in \Sigma^*$ is a substring in the word $w \in \Sigma^*$.

Note2: You may freely use results from the lectures and recitations.

1. Let L be a language over $\{0, 1, ..., 9\}$ such that $w \in L$ iff w is an i.d. number of a member of the Knesset of 2020. Prove that L is regular.

2. Determine whether the following languages are regular. Prove your answer.

   (a) $L_1 = \{w \mid \#_a(w) \geq \#_b(w)\}$ over $\Sigma = \{a, b, c\}$.

   (b) $L_2 = \{w \mid |w| \in \mathbb{N}_{even} \ \wedge \ w = w^R\}$ over $\Sigma = \{(,)\}$.

   (c) $L_3 = \{w \mid |w| \in \mathbb{N}_{even} \ \wedge \ w = w^R\}$ over $\Sigma = \{(\}$.

   (d) $L_4 = \{w \mid \exists n \in \mathbb{N} \ s.t. \ |w| = n^3\}$ over $\Sigma = \{1\}$.

3. Determine whether the following languages are context free. Prove your answer.

   (a) $L_1 = \{a^{2n}b^{3n} | n \in \mathbb{N}\}$ over $\Sigma = \{a, b\}$.

   (b) $L_2 = \{a^n b^m c^{n+m} | n \in \mathbb{N}\}$ over $\Sigma = \{a, b, c\}$.

   (c) $L_3 = \{x\#y | x, y \in \{0, 1\}^* \wedge x \ is \ a \ substring \ of \ y\}$ over $\Sigma = \{0, 1, \#\}$.

   (d) $L_4 = \{x_1\#x_2\#...\#x_n \mid \forall i.x_i \in \{0, 1\}^* \wedge n \geq 2 \wedge \exists i \neq j.x_i = x_j\}$ over $\Sigma = \{0, 1, \#\}$.

4. Let $Inv(L) = \{xyz \mid xy^R z \in L\}$. Prove or disprove:

   - The regular languages are closed under this operation.
   - The context free languages are closed under this operation.

5. Prove that a language over an unary alphabet ($|\Sigma| = 1$) is regular iff it is context free.
   Hint: Use the pumping lemma.

6. Find a context free grammar for $\overline{L}$ where $L = \{ww \mid w \in \{0,1\}^*\}$. What does it imply about context free languages?

7. This question deals with algorithmic problems.

   (a) Describe an algorithm that given a DFA A, decides if $L(A)$ is infinite. (possible hint: use the pumping lemma).

   (b) Describe an algorithm that given a DFA A, decides if $|L(A)| = 9,122,009$.

   (c) Describe an algorithm that given a CFG G, decides if $L(G)$ is infinite.