Exercise 3 - Computational Models

Submission date and time: 05/05/2011, 16:00

Your assignment should be in box 303 before 16:00 !

1. Define a binary operation on languages over the same alphabet Σ:

$$L_1/L_2 = \{x \in \Sigma^* \mid \exists y \in L_2 . xy \in L_1\}$$

   (a) Let $L_1 = \{0^n 1^n \mid n \in \mathbb{N}\}$ and $L_2 = \mathcal{L}(1^*)$. What is L_1/L_2?

   (b) Let $L_1 = \mathcal{L}((01)^*)$ and $L_2 = \mathcal{L}((0 \cup 1)^*0)$. What is L_1/L_2?

   (c) Prove or disprove:

      i. If L_1 is a regular language and L_2 is a regular language then $L1/L_2$ is a regular language.

      ii. If L_1 is a regular language then $L1/L_2$ is a regular language.

      iii. If L_1 is a context free language then $L1/L_2$ is a context free language.

      iv. If L_1 is a context free language and L_2 is a regular language then $L1/L_2$ is a context free language.

2. Write a 1-tape Turing machine (TM) that decides (i.e., halts for any input with the correct answer) $L = \{w \in \{0,1\}^* \mid w \textit{ does not contain } 00\}$. Give a formal description, including the transition function.

3. A TM *computes a function* $f : \Sigma^* \to \Sigma^*$ if it halts with $f(x)$ on its tape, when given x as input. Write a 1-tape TM that computes $f(w) = w^R$ (the reverse of $w \in \{0,1\}^*$). Give a formal description, including the transition function.

4. Write a 1-tape TM that decides (i.e., halts for any input with the correct answer) $L = \{w \in \{0,1\}^* \mid \#_0(w) \neq 2 \cdot \#_1(w)\}$. Describe the way the TM work. There is no need to specify the transition function.

5. A *searching TM* is a TM that has also a search command. Its transition function δ is from $Q \times \Gamma$ to $Q \times \Gamma \times \{L, R\} \times \{\Gamma \cup \text{“}None\text{”}\}$. $\delta((q, a)) =$

(q', a', L, a'') (or R instead of L) means that if the machine is in state q, and the current cell contains a, then it will write a' instead of it, and the head will move left (or right) till it sees a'', or it reaches the end of the tape. Once it stops moving the state of the TM will be changed to q'. The search is optional: $\delta((q, a)) = (q', a', L, \text{“}None\text{”})$ means that we want usual transition, and no search would be done.

Determine whether the class of languages decided by searching TMs is R. Prove your claims, and give formal specifications of the TMs in your answer.

6. Prove or disprove:

   (a) R is closed under complementation.

   (b) $R.E.$ is closed under complementation.

   (c) $R.E.$ is closed under intersection.

   (d) $co - R.E.$ is closed under intersection.

   (e) $R.E$ is closed under Kleene star.

7. A researcher claims that he has detected for every alpahbet Σ a finite set of important TMs that can decide any decidable language over Σ.

   (a) Prove that he is wrong.

   (b) Let A be the set of languages decided by TMs that have at most 1000 states and at most 1000 tape symbols (in addition to the symbols of Σ and $\sqcup$). Prove that $A \neq R$.