Exercise 4 - Computational Models - Spring 2011

Submission date and time: 19/5/2011, 15:00. Please submit directly to box 311 in Schreiber. Mariano will also collect the exercises in his recitations

1. Let L be any infinite language in $R.E.$. Prove that L has an enumerator f such that f is monotone on the odd numbers, i.e. for every two odd numbers a, b, if $a < b$ then $f(a) < f(b)$.

2. Are the following languages decidable? Prove your answers. Don't use Rice Thm.

   (a) $\{\langle M\rangle | L(M) \textit{ is infinite}\}$

   (b) $\{\langle M_1, M_2\rangle || L(M_1)| \leq |L(M_2)|\}$

   (c) $\{\langle M\rangle | \textit{ there exists an input that } M \textit{ accepts in less then 100 steps}\}$

3. Refute: For every infinite language L, there exists an infinite decidable language $L' \subseteq L$.

4. The $3x + 1$ problem concerns the function f, which takes odd integers n to $3n + 1$ and even integers n to $n/2$. If you start with an integer n and iterate f, you obtain a sequence x, f(x), f(f(x)),... Stop if you ever hit 1. For example, if $n = 17$, you get the sequence 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1. Extensive computer tests have shown that every starting point between 1 and a large integer gives a sequence that ends in 1. But, the question of whether all positive starting points end up at 1 is unresolved; it is called the $3x + 1$ problem. Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that prints the answer to the $3x + 1$ problem.

5. Prove or disprove:

   (a) $A_{CFG} \leq A_{DFA}$.

   (b) $EMPTY_{DFA} \leq_m ALL_{DFA}$.

   (c) $Halt_{TM} \leq A_{CFG}$.

   (d) $A_{TM} \leq EMPTY_{TM}$.

(e) $L(0^*1^*) \leq_m A_{TM}$.

(f) $\emptyset \leq A_{TM}$.

(g) if $L \in R.E.$, then $L \leq_m A_{TM}$.

(h) $\leq_m$ is transitive relation.

Note: $L_1 \leq L_2$ means that there is a reduction from L_1 to L_2, i.e. there is a TM that decides L_1 that in addition to the usual operation can also "ask" questions of the form "is $x \in L_2$?". $L_1 \leq_m L_2$ means there is a *mapping* reduction from L_1 to L_2, as defined in the lectures.