

Exercise 5 - Computational Models - Spring 2010/2011

Submission date and time: 02/06/2011, 16:00

Please submit directly to box 303 in Schreiber. Mariano will also collect the exercises in his recitations.

1. For the following decision problems determine whether they belong to \mathcal{R} , $\mathcal{RE} \setminus \mathcal{R}$, $\text{co-}\mathcal{RE} \setminus \mathcal{R}$ or none of the above:
 - (a) Input: Turing machine M
Question: is there an x for which M halts?
 - (b) Input: Turing machine M
Question: is there life beyond earth ?
 - (c) Input: Turing machine M and inputs x and y
Question: does M halt on exactly one of the inputs?
 - (d) Input: Turing machine M
Question: is $|L(M)| \geq 3$?
 - (e) Input: Turing machine M
Question: is $L(M) \in \mathcal{RE}$?
 - (f) Input: Turing machine M such that $|\langle M \rangle| < 10^{100}$
Question: does M halt on ϵ ?
 - (g) Input: Turing machine M
Question: Is it true that for all inputs x , M 's run over x never reaches position $|x| + 7$ on the tape?
 - (h) Input: Turing machine M whose transition function is in $Q \times \Gamma \rightarrow Q \times \Gamma \times \{R\}$ (i.e. TM that can only move right), and an input x
Question: Does M halt when it runs on x ?
 - (i) Input: Turing machine M and a number k
Question: Does M halt on all inputs of length at most k^2 in k^3 steps?
 - (j) Input: LBAs A_1, A_2
Question: Does $L(A_1) = L(A_2)$?

2. Let $L_1, L_2 \in \mathcal{RE} \setminus \mathcal{R}$. Prove whether the following is possible:
- (a) $L_1 \cup L_2 \in \mathcal{R}$
 - (b) $L_1 \cup L_2 \in \mathcal{R}$ and $L_1 \cap L_2 \in \mathcal{R}$
3. Let $A \in \mathcal{RE} \setminus \mathcal{R}$ over $\Sigma = \{0, 1\}$. Let $B = \{0w \mid w \in A\} \cup \{1w \mid w \notin A\}$. Prove that $B \notin \mathcal{RE} \cup \text{co}\mathcal{RE}$.