

Exercise 6 - Computational Models - Spring 2011

Submission date and time: 16/6/2011, 15:00. Please submit to box 311 in Schreiber.

1. Show that $A_{TM} \in \text{NP-hard}$
2. Show that if $\text{NPC} \cap \text{Co-NP} \neq \emptyset$ then $\text{NP} = \text{Co-NP}$
3. For the following decision problems, determine whether they are in P or in NPC. Prove your answer.
 - (a) Input: sets A_1, \dots, A_n , and a number k .
Question: do there exist k mutually disjoint sets A_{i_1}, \dots, A_{i_k} ?
 - (b) Input: sets A_1, \dots, A_n , and a number k .
Question: do there exist k sets A_{i_1}, \dots, A_{i_k} such that $A_{i_j} \cap A_{i_l} \neq \emptyset$ for every $j \neq l$?
 - (c) Input: numbers $w_1, \dots, w_n, v_1, \dots, v_n, w, v$.
Question: does there exist $S \subseteq \{1, \dots, n\}$ s.t. $\sum_{i \in S} w_i \leq w$ and $\sum_{i \in S} v_i \geq v$?
 - (d) Input: graph $G = (V, E)$.
Question: can G 's vertices be coloured in 4 colours, such that for every $(u, v) \in E$, u and v have different colours?
 - (e) Input: a 3CNF formula ψ .
Question: does there exist an assignment that satisfies ψ and gives *True* for exactly 10 variables?
 - (f) Input: a 3CNF formula ψ with even number of variables.
Question: does there exist an assignment that satisfies ψ and gives *True* for exactly one half of the variables?
 - (g) Input: graph G , a number k .
Question: does there exist a simple path in G of length $\geq k$?
 - (h) Input: graph G with an odd number of vertices.
Question: Does either $|MAX - IS(G)|$ is even or $|MIN - VC(G)|$ is even?
where $|MAX - IS(G)|$ is the size of a maximal independent set in G , and $|MIN - VC(G)|$ is the size of a minimal vertex cover in G